Problem 1. Write the statements (a)-(e) in terms of the letters h = "Jon is healthy", w = "Jon is wealthy", s = "Jon is wise", and the symbols ¬, ∧, ∨, ⊕.

Part (a) Jon is healthy and wealthy but not wise.
$\boxed{h \wedge w \wedge \neg s}$

Part (b) Jon is not wealthy but he is healthy and wise.
$\boxed{\neg w \wedge h \wedge s}$

Part (c) Jon is neither healthy, wealthy, nor wise.
Answer 1: = "Jon is not (either healthy, wealthy, or wise)" = $\boxed{\neg(h \vee w \vee s)}$

Answer 2: If Jon is neither, then he is none of these things, so he is not healthy and not wealthy and not wise:
$\boxed{\neg h \wedge \neg w \wedge \neg s}$

In fact, Answer 1 simplifies to Answer 2 by de Morgan's law.

Part (d) Jon is neither wealthy nor wise, but he is healthy.
Answer: = "Jon is not (either wealthy or wise), and he is healthy" = $\boxed{\neg(w \vee s) \wedge h}$

Part (e) Jon is wealthy, but he is not both healthy and wise.
Answer: = "Jon is wealthy, but he is not (both healthy and wise)" = $\boxed{w \wedge \neg(h \wedge s)}$

Problem 2. Write the statements (a)-(c) in terms of the letters p = "x > 5", q = "x = 5", r = "10 > x", and the symbols ¬, ∧, ∨, ⊕.

Part (a) x ≥ 5
Verbally, we read this as "x is greater than OR equal to 5", which in longer form reads as "x is greater than 5 or x is equal to 5".
Answer: $\boxed{p \vee q}$
Answer 2: since equality and strict inequality cannot occur at the same time, we notice that the two statements

- "$x > 5$" OR "$x = 5$"
- "$x > 5$" XOR "$x = 5$" (here, XOR stands for "exclusive-or" aka $\oplus$)
  are logically equivalent for all real numbers x so another way to write "x ≥ 5 " is as $p \oplus q$.

Part (b) 10 > x > 5
We read this as "10 is greater than x and x is greater than 5".
Answer: $\boxed{r \wedge p}$

Part (c) 10 > x ≥ 5
Answer: $\boxed{r \wedge (p \vee q)}$.

Problem 3. True or false:
Part (a) "Either 12321 ≡ 0 (mod 3) or 12321 ≡ 0 (mod 9)".

We know that $12321 \equiv 1+2+3+2+1 = 9 \equiv 0 \pmod 3$ and likewise $\equiv 0 \pmod 9$ so

"12321 ≡ 0 (mod 3)" or "12321 ≡ 0 (mod 9)" evaluates to TRUE OR TRUE = TRUE.

Part (b) "12321 ≡ 0 (mod 3)" ⊕ "12321 ≡ 0 (mod 9)"

"12321 ≡ 0 (mod 3)" ⊕ "12321 ≡ 0 (mod 9)" evaluates to TRUE ⊕ TRUE = FALSE since exclusive-or is true precisely when exactly one of the argument is true.

Problem 4. Show $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$

We write the truth table of $\neg(p \wedge q)$ out:

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

We also write the truth table of $\neg p \vee \neg q$ out:

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Because the columns for $\neg(p \wedge q)$ and $\neg p \vee \neg q$ have identical T and F values, $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

Problem 5. Show $(p \wedge q) \vee r \not\equiv p \wedge (q \vee r)$.

We can either write the truth tables of a and $p \wedge (q \vee r)$ out and see their final columns are not identical, or find some truth values of p, q, r to plug in to make them not equal. Let's try $p = F, q = F, r = T$:

$$(p \wedge q) \vee r = (F \wedge F) \vee T = T, \quad \text{while} \quad p \wedge (q \vee r) = F \wedge (F \vee T) = F \wedge T = F$$

so $(p \wedge q) \vee r \not\equiv p \wedge (q \vee r)$.